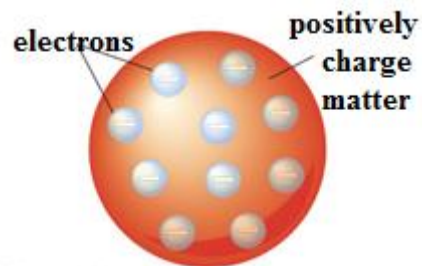


1. **Thomson's Atomic Model:** First atomic model was given by Sir Joseph John Thomson in 1898 also known as *Plum Pudding Model* or *Watermelon Model*.

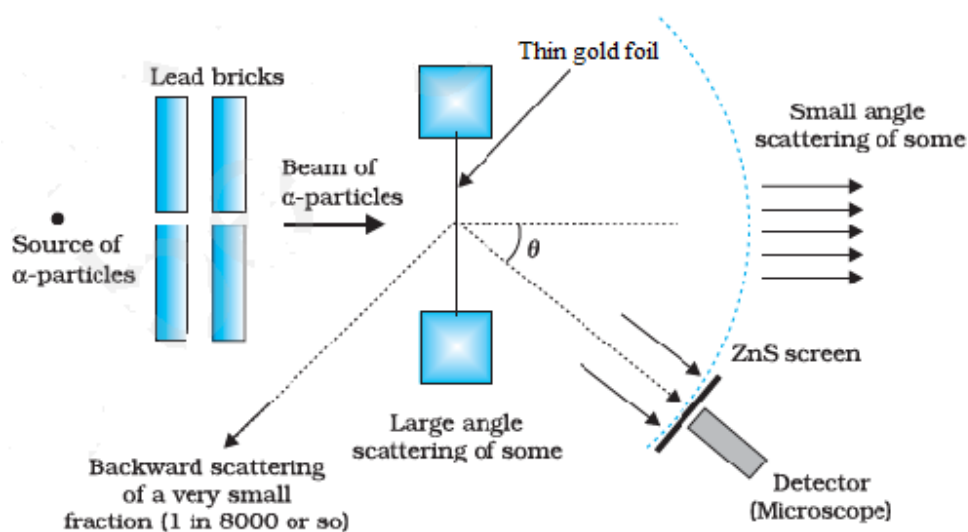
According to Thompson atom is a like watermelon. The positively charged matter inside atom is like reddish part of watermelon and electrons are uniformly embedded like seeds in watermelon. Atom is a sphere of **radius 10^{-10} m** and **electrically neutral**. He also discovered the *electron*, a negatively charged part of every atom.



Limitation: It could not explain the origin of spectral series of H atom and also result of Rutherford α -Particle Scattering Experiment.

2. **Rutherford α -Particle Scattering Experiment:**

This experiment was carried out by H. Geiger and E. Marsden in 1911 on suggestion of Ernst Rutherford. They directed a beam of 5.5 MeV α -particles (α -particle is a nucleus of helium atom carrying a charge of '+2e' and mass equal to 4 times that of hydrogen atom) emitted from a $^{214}\text{Bi}_{83}$ radioactive source at thin metal foil (10^{-7} m) made of gold. The scattered α -particles were observed through a rotatable detector consisting of ZnS screen and a microscope.



Schematic arrangement of the Geiger-Marsden experiment

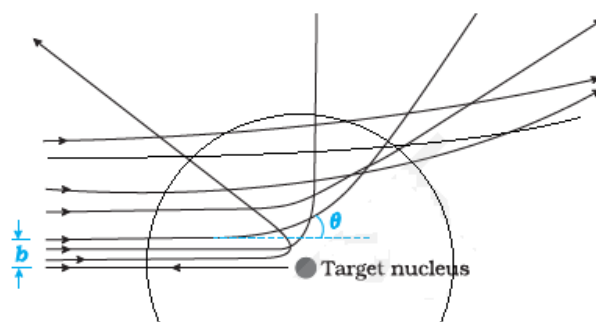
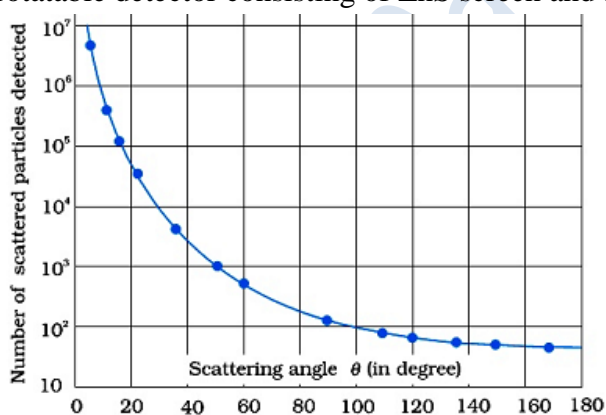


FIGURE Trajectory of α -particles in the coulomb field of a target nucleus. The impact parameter, b and scattering angle θ

The explanation of above results led to the birth of **Rutherford's planetary model of atom/Nuclear model of the atom**.

According to this model the entire positive charge and most of the mass of the atom is concentrated in a very small region called the **nucleus** with electrons revolving around the nucleus just as planets revolve around the sun.

Observation and Conclusion:

- (i) Most of α -particle pass straight though gold foil or suffer only small deflection \rightarrow most of the space within atom must be empty.

- (ii) A very few α -particle about 1 in 8000 gets deflected through large angle, even more than $90^\circ \rightarrow$ all the positive charge and mass of the atom is concentrated in very small region **called nucleus**.
- (iii) An α -particle gets rebounded from the gold foil.

2.1-Distance of Closest Approach (Estimation of size of Nucleus): It may be defined as distance between the centre of the nucleus and the point from which an α -particle approaching directly to the nucleus returns.

Consider an α -particle of mass m moving with velocity v and approaches to nucleus. It experiences Coulombic repulsion and its kinetic energy gets converted into electric potential energy. At a certain distance from the nucleus, α -particle stops and retrace its path this distance (r_o) is called *Distance of Closest Approach*.

$$\text{Kinetic energy of } \alpha\text{-particle (K)} = \frac{1}{2} mv^2$$

$$\text{Electrostatic P.E. of } \alpha\text{-particle (U)} = \frac{k q_1 q_2}{r_o} = \frac{k 2e \cdot Ze}{r_o}$$

From energy conservation, $K = U$

$$\frac{1}{2} mv^2 = \frac{k 2 Ze^2}{r_o}$$

$$r_o = \frac{4 k Ze^2}{mv^2} = \frac{2 k Ze^2}{K}$$

In this experiment K (K.E.) = 5.5 MeV = $5.5 \times 10^6 \times 1.6 \times 10^{-19}$ J

For Gold Atomic number (Z) = 79, k (Coulomb's constant) = 9×10^9 Nm²/C²

$$r_o = \frac{2 k Ze^2}{K} = \frac{2 \times 9 \times 10^9 \times 79 \times (1.6 \times 10^{-19})^2}{5.5 \times 10^6 \times 1.6 \times 10^{-19}}$$

$$r_o = 4.13 \times 10^{-14} \text{ m} = 41.3 \text{ fm}$$

From estimating distance of closest approach above, we may conclude that radius of the nucleus is of the order of one Fermi.

2.2- Impact Parameter: The perpendicular distance of the velocity vector of the α -particle from the line passing to centre of the nucleus, when it is far away from the atom.

The shape of the trajectory of the scattered α -particle depends on the impact parameter.

$$\text{Impact parameter (b)} = \frac{Ze^2 \cot \frac{\theta}{2}}{4\pi\epsilon_0 K}$$

Conclusion (significance) from study of impact parameter,

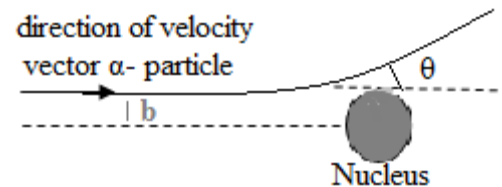
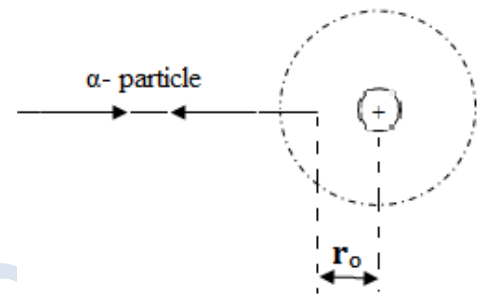
i) For large value of b , $\cot \frac{\theta}{2}$ is large and θ , the scattering angle is small. i.e. α -particles travelling far away from the nucleus suffer small deflections.

ii) For small value of b , $\cot \frac{\theta}{2}$ is also small and θ , the scattering angle is large. i.e. α -particles travelling close to the nucleus suffer large deflections.

iii) For $b = 0$ i.e. α -particles directed towards the centre of the nucleus, $\cot \frac{\theta}{2} = 0$ or $\frac{\theta}{2} = 90^\circ$ or $\theta = 180^\circ$ -The α -particles retrace their path.

2.3- Rutherford's Atomic Model:

- An atom consists of a small and massive core called *Nucleus*, in which the entire positive charge and almost the whole mass of the atom are concentrated.
- The size of the nucleus ($\approx 10^{-15}$ m) is very small as compared to the size of the atom ($\approx 10^{-10}$ m).
- The nucleus is surrounded by a suitable number of electrons so that their total negative charge is equal to the total positive charge on the nucleus and the atom as a whole is electrically neutral.



(iv) The electrons revolve around the nucleus in various orbits just as planets revolve around the sun. The centripetal force required for their revolution is provided by the electrostatic attraction between the electron and the nucleus.

2.4- Limitations of Rutherford's Atomic Model:

- i. Rutherford's atomic model cannot explain the stability of an atom.
- ii. In Rutherford's atomic model an electron can revolve in orbits of all possible radii .so it should emit a continuous spectrum. But an atom like hydrogen always emits a discrete line spectrum.

3. Bohr's Atomic Model: In 1913 Bohr's suggested a new model of atom which overcomes drawback of Rutherford's atomic model.

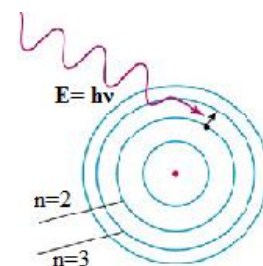
3.1- POSTULATES:

- 1) **Nuclear concept-** An atom consists of a small and massive central core, called *Nucleus* around which electrons revolve. The centripetal force required for their revolution is provided by the electrostatic attraction between the electrons and nucleus.
- 2) **Quantum condition-** The electrons are permissible to revolve only in those orbits in which the angular momentum of an revolving electron is integral multiple of $h/2\pi$; where h is Plank's constant. Therefore, for any permitted orbit,

$$L = mvr = \frac{nh}{2\pi} \quad n = 1, 2, 3$$

- 3) **Stationary orbits-** While revolving in the permissible orbits, an electron does not radiate energy such non-radiating orbits are called *Stationary Orbits*.
- 4) **Frequency condition-**An atom can emit or absorb radiation in the form of discrete energy photons only when an electron jumps from a higher to a lower orbit or from a lower to a higher orbit, respectively. If E_1 and E_2 are the energies associated with these permitted orbits then the frequency ν of the emitted or absorbed radiation is given by

$$h\nu = E_2 - E_1$$



3.2-Bohar's Theory of Hydrogen Atom: According to Bohr's theory of hydrogen atom consists of nucleus with a positive charge Ze and a single electron of charge e , which revolves around it in circular orbit of radius r .

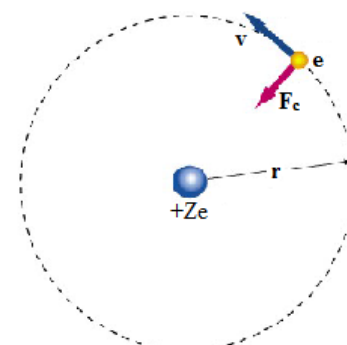
3.2.1 Quantization of Radius: Consider an electron of mass m and charge e is revolving in circular orbit around nucleus, a centripetal force (F_c) must be acting on it which is provided by electrostatic force (F_e) between nucleus and electron.

$$F_c = F_e$$

$$\frac{mv^2}{r} = \frac{k e Ze}{r^2}$$

$$mv^2 = \frac{k Ze^2}{r} \dots\dots\dots(1)$$

$$r = \frac{k Ze^2}{mv^2} \dots\dots\dots(2)$$



The Bohr Atom

From Bohr's quantisation condition of angular momentum (L),

$$L = mvr = \frac{nh}{2\pi}$$

$$r = \frac{nh}{2\pi mv} \dots\dots\dots(3)$$

Comparing equation (2) and equation (3),

$$\frac{k Ze^2}{mv^2} = \frac{nh}{2\pi mv}$$

$$v = \frac{2\pi k Ze^2}{nh} \dots\dots\dots(4)$$

Putting this value of v in equation (3),

$$r_n = r = \frac{n^2 h^2}{4\pi^2 m k Z e^2} \dots\dots\dots(5)$$

$$r_n = \frac{(6.6 \times 10^{-34})^2}{4 \times (3.14)^2 \times 9.1 \times 10^{-31} \times 9 \times 10^9 \times (1.6 \times 10^{-19})^2} \cdot \frac{n^2}{Z}$$

$$r_n = 5.29 \times 10^{-11} \cdot \frac{n^2}{Z} \quad \text{m}$$

$$r_n = 0.529 \cdot \frac{n^2}{Z} \quad \text{\AA}$$

The mean radius of the orbit of an electron revolving around nucleus of H-atom in its ground state ($n=1$) is called Bohr's radius and its value is 0.529 Å

3.3-Quantization of Velocity: From equation (3) orbital speed of electron is,

$$v = \frac{2\pi k Z e^2}{nh}$$

Multiplying by $\frac{c}{c}$ in RHS, $c =$ speed of light $v = \frac{2\pi k Z e^2}{nh} \cdot \frac{c}{c}$

Rearranging the above equation, For Hydrogen atom $Z = 1$ $v = \frac{2\pi k 1 e^2}{ch} \cdot \frac{c}{n}$
 where, $\frac{2\pi k e^2}{ch} = \frac{1}{137}$ is called *Fine structure constant* and it is dimensionless.

So above equation becomes, $v = \frac{1}{137} \cdot \frac{c}{n} \quad \text{m/s}$

3.4-Quantization of Energy: Total energy of the electron revolving around nucleus is equal to sum of kinetic energy and potential energy.

Kinetic energy of the electron is, $\text{K.E.} = \frac{1}{2} m v^2$

From equation (1), $m v^2 = \frac{k Z e^2}{r}$ putting this value in above equation

$$\text{K.E.} = \frac{1}{2} m v^2 = \frac{k Z e^2}{2r}$$

Potential energy of the electron in n^{th} orbit is

$$\text{P.E.} = k q_1 q_2 / r = \frac{k Z e(-e)}{r} = - \frac{k Z e^2}{r}$$

Hence total energy of the electron in n^{th} orbit is,

$$E_n = \text{K.E.} + \text{P.E.}$$

$$E_n = \frac{k Z e^2}{2r} - \frac{k Z e^2}{r} = - \frac{k Z e^2}{2r}$$

Putting value of 'r' from equation (5), $E_n = \frac{-k Z e^2}{2 \times \frac{n^2 h^2}{4\pi^2 m k Z e^2}}$

$$E_n = - \frac{2\pi^2 m k^2 Z^2 e^4}{n^2 h^2} \dots\dots\dots(6)$$

Here *negative sign indicate that electron is bound to nucleus and energy is required to make it free.*

When electron is in first orbit ($n=1$), the corresponding energy of electron is called Ground energy level, Energy in the second orbit ($n=2$) is called First excited energy level, Energy in the third orbit ($n=3$) is called Second excited energy level.

When electron jump from higher energy orbit (outer) to lower energy orbit (inner), energy is emitted in the form of radiation called *emission spectrum*. When energy is given from outside electron jump from lower energy orbit to higher energy orbit, the absorbed energy spectrum is called *absorption spectrum*. In absorption only one series is possible while in emission electron may jump from any state to lower state, so in emission all series are possible.

3.5-Spectral Series of Hydrogen Atom:

From Bohr's Theory energy of an electron in n^{th} orbit is given by,

$$E_n = -\frac{2\pi^2 m k^2 Z^2 e^4}{n^2 h^2}$$

According to Bohr's Theory (*frequency condition*) when an electron jump from higher energy level (n_2) to lower energy level (n_1), energy is released in the form of radiation of frequency ν or wavelength λ ,

$$E = h\nu = E_{n_2} - E_{n_1}$$

$$h\nu = \frac{2\pi^2 m k^2 Z^2 e^4}{h^2} \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

$$\nu = \frac{2\pi^2 m k^2 Z^2 e^4}{h^3} \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

We know that $\nu = \frac{c}{\lambda}$, putting this value in above equation,

$$\frac{c}{\lambda} = \frac{2\pi^2 m k^2 Z^2 e^4}{h^3} \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

$$\text{Wave number } (\bar{\nu}) = \frac{1}{\lambda} = \frac{2\pi^2 m k^2 Z^2 e^4}{c h^3} \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

or $\bar{\nu} = \frac{1}{\lambda} = R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$ Rydberg's Formula, this relation explains spectral series of Hydrogen atom which arises due to transition of electron from various outer orbits to inner orbit.

Where $R = \frac{2\pi^2 m k^2 Z^2 e^4}{c h^3}$ *Rydberg constant* and its value is $1.0973 \times 10^7 \text{ m}^{-1}$.

Various *spectral series of Hydrogen atom* are given below which origin can be explained on the basis of Bohr's theory,

- i) **Lyman series.** If an electron jumps from any higher energy level $n_2 = 2, 3, 4, \dots, \infty$ to a lower energy level $n_1 = 1$, we get a set of spectral lines called *Lyman Series* which belong to the *Ultraviolet Region* of the electromagnetic spectral. Wave number associated with this series is given by

$$\bar{\nu} = \frac{1}{\lambda} = R \left[\frac{1}{1^2} - \frac{1}{n_2^2} \right] \quad n_2 = 2, 3, 4, \dots$$

Longest wavelength of Lyman series is obtained when $n_2 = 2$

$$\bar{\nu} = \frac{1}{\lambda} = R \left[\frac{1}{1^2} - \frac{1}{2^2} \right] = R \left[\frac{1}{1^2} - \frac{1}{4} \right] = \frac{3R}{4}$$

$$\lambda_{\max} = \frac{4}{3 \times 1.0973 \times 10^7} = 1215 \text{ \AA}$$

Shortest wavelength of Lyman series is obtained when $n_2 = \infty$

$$\bar{\nu} = \frac{1}{\lambda} = R \left[\frac{1}{1^2} - \frac{1}{\infty^2} \right] = R \left[\frac{1}{1^2} - 0 \right] = R$$

$$\lambda_{\min} = \frac{1}{1.0973 \times 10^7} = 911.6 \text{ \AA}$$

- ii) **Balmer series.** The spectral series corresponding to the transitions $n_2 = 3, 4, 5, \dots$ to $n_1 = 2$, lies in the *Visible Region* and is called *Balmer Series*.

$$\bar{\nu} = \frac{1}{\lambda} = R \left[\frac{1}{2^2} - \frac{1}{n_2^2} \right] \quad n_2 = 3, 4, 5, \dots$$

Longest wavelength of Balmer series is obtained when $n_2 = 3$ $\lambda_{\max} = 6563 \text{ \AA}$

Shortest wavelength of Balmer series is obtained when $n_2 = \infty$ $\lambda_{\min} = 3646 \text{ \AA}$

- iii) **Paschen series.** If $n_2 = 4, 5, 6, \dots$ and $n_1 = 3$, we get a spectral series in the *Infrared Region* which is called *paschen series*.

$$\bar{\nu} = \frac{1}{\lambda} = R \left[\frac{1}{3^2} - \frac{1}{n_2^2} \right] \quad n_2 = 4, 5, 6, \dots$$

Longest wavelength of Paschen series is obtained when $n_2 = 4$ $\lambda_{\max} = 18752 \text{ \AA}$

Shortest wavelength of Paschen series is obtained when $n_2 = \infty$ $\lambda_{\min} = 8204 \text{ \AA}$

- iv) **Brackett series.** If $n_2 = 5, 6, 7, \dots$ and $n_1 = 4$, we get a spectral series in the *Infrared Region* which is called *Brackett Series*.

$$\bar{\nu} = \frac{1}{\lambda} = R \left[\frac{1}{4^2} - \frac{1}{n_2^2} \right] \quad n_2 = 5, 6, 7, \dots$$

Longest wavelength of Bracket series is obtained when $n_2 = 5$ $\lambda_{\max} = 40515 \text{ \AA}$

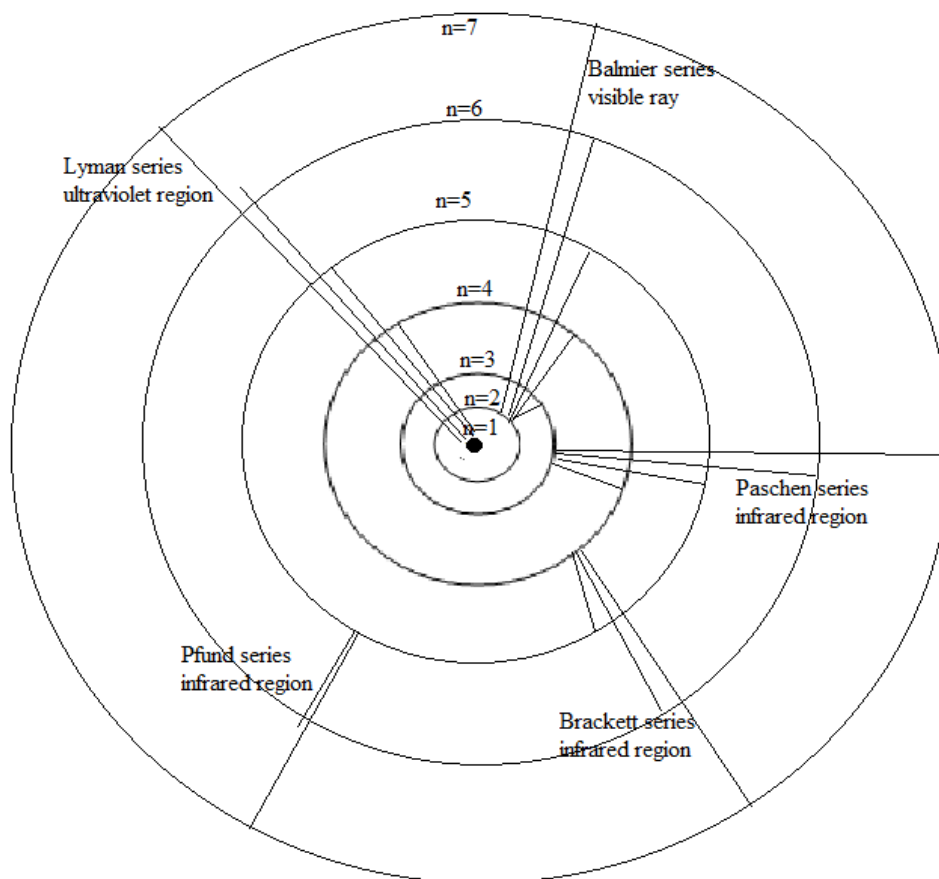
Shortest wavelength of Bracket series is obtained when $n_2 = \infty$ $\lambda_{\min} = 14585 \text{ \AA}$

v) **Pfund series.** If $n_2 = 6, 7, 8, \dots$ and $n_1 = 5$, we get a spectral series in the *Infrared Region* which is called *Pfund Series*.

$$\bar{\nu} = \frac{1}{\lambda} = R \left[\frac{1}{5^2} - \frac{1}{n_2^2} \right] \quad n_2 = 6, 7, 8, \dots$$

Longest wavelength of Pfund series is obtained when $n_2 = 6$ $\lambda_{\max} = 74584 \text{ \AA}$

Shortest wavelength of Pfund series is obtained when $n_2 = \infty$ $\lambda_{\min} = 22789 \text{ \AA}$



Excitation energy: The energy required by an electron to jump from the ground state to any excited state is called excitation energy.

First excitation energy of hydrogen = $E_2 - E_1 = -3.4 - (-13.6) = 10.2 \text{ eV}$

Second excitation energy of hydrogen = $E_3 - E_1 = -1.51 - (-13.6) = 12.09 \text{ eV}$

Excitation potential: The accelerating potential, which gives energy to an electron to jump from inner orbit to an outer orbit, is called excitation potential.

First excitation potential of hydrogen = $-3.4 - (-13.6) = 10.2 \text{ V}$

Second excitation potential of hydrogen = $-1.51 - (-13.6) = 12.09 \text{ V}$

Ionisation energy: The energy required by an electron to just escape it completely from its orbit is called ionisation energy.

Ionisation energy of hydrogen = $E_\infty - E_1 = 0 - (-13.6) = 13.6 \text{ eV}$

Ionisation potential: The accelerating potential, which gives energy to an electron to escape from atom completely, is called ionisation potential.

First excitation potential of hydrogen = $0 - (-13.6) = 13.6 \text{ V}$

3.5.2- Energy Level Diagram for Hydrogen: It is a diagram in which the energies of the different stationary orbits of an atom are represented by parallel horizontal lines, drawn according to some suitable energy scale.

From Bohr's theory, the total energy of an electron in nth orbit is given by,

$$E_n = -\frac{2\pi^2 mk^2 Z^2 e^4}{n^2 h^2}$$

For hydrogen ($Z = 1$) energy of the electron in the first orbit ($n = 1$) is given by,

$$\begin{aligned} E_1 &= 2 \times (3.14)^2 \times (9.11 \times 10^{-31}) \times \\ & (9 \times 10^9)^2 \times (1.6 \times 10^{-19})^4 / (6.63 \times 10^{-34})^2 \cdot 1^2 \\ &= -21.76 \times 10^{-19} \text{ J} \\ &= -21.76 \times 10^{-19} / 1.6 \times 10^{-19} \text{ eV} \\ &= -13.6 \text{ eV} \end{aligned}$$

Hence we may write,

$$E_n = E_1/n^2 = -13.6/n^2 \text{ eV}$$

$$E_1 = -13.6/1^2 = -13.6 \text{ eV}$$

$$E_2 = -13.6/2^2 = -3.4 \text{ eV}$$

$$E_3 = -13.6/3^2 = -1.51 \text{ eV}$$

$$E_4 = -13.6/4^2 = -0.85 \text{ eV}$$

$$E_5 = -13.6/5^2 = -0.54 \text{ eV}$$

Thus, an electron can have only

certain definite values of energy while revolving in different orbits. This is called **energy quantization**.

3.6-Limitations of Bohr's Theory:

- 1) This theory is applicable only to hydrogen-like single electron atoms and fails in the case of atoms with two or more electrons.
- 2) It does not explain why only circular orbits should be chosen when elliptical orbits are also possible.
- 3) Bohr's theory does not tell anything about the relative intensities of the various spectral lines. Bohr's theory predicts only the frequencies of these lines.

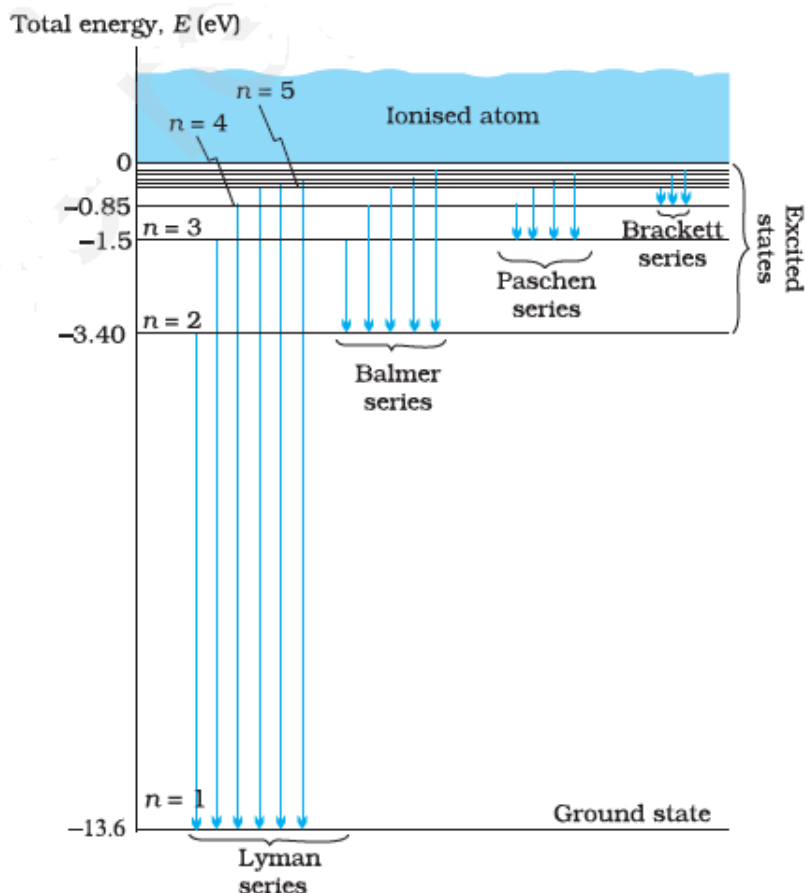


Fig. Energy level diagram of hydrogen Atom

Nucleons: Protons and neutrons which are present in nuclei are collectively known as nucleons.

Atomic number (Z): The total number of protons in the nucleus is equal to atomic number.

Mass number (A): The total number of protons and neutrons present in a nucleus.

$$\text{Number of neutrons in an atom} = N = A - Z$$

Nuclear mass: The total mass of the protons and neutrons present in a nucleus.

Nuclide: A nuclide is a specific nucleus of an atom characterized by its atomic number Z and mass number and represented as ${}^A_Z\text{X}$.

Isotopes: The atoms of an element which have the same atomic number but different mass number are called isotopes. Hydrogen has three isotopes: Hydrogen protium (${}^1_1\text{H}$) – its nucleus has one proton; deuterium (${}^2_1\text{H}$)–its nucleus has one proton and one neutron; and tritium (${}^3_1\text{H}$) –its nucleus has one proton and two neutrons.

Isobars: The atoms having the same mass number but different atomic number.

- 1) ${}^{37}_{17}\text{Cl}$ and ${}^{37}_{16}\text{S}$, as both have same $A = 37$.
- 2) ${}^{40}_{20}\text{Ca}$ and ${}^{40}_{18}\text{Ar}$, as both have same $A = 40$.

Isotones: The nuclides having the same number of neutrons are called isotones.

- 1) ${}^{37}_{17}\text{Cl}$ and ${}^{39}_{19}\text{K}$ are isotones, $N = A - Z = 20$.
- 2) ${}^{198}_{80}\text{Hg}$ and ${}^{197}_{79}\text{Au}$ are isotones, $N = A - Z = 118$

Atomic masses (u): Masses of atoms, nuclei etc. are expressed in terms of atomic mass unit, and it may be defined as 1/12th of the actual mass of carbon-12 atom.

$$1 \text{ u} = 1.660565 \times 10^{-27} \text{ kg}$$

Nuclear size: Experimental observations show that the volume (V) of a nucleus is directly proportional to its mass number (A),

$$V \propto A$$

As nucleus is spherical in shape so, $V = \frac{4\pi R^3}{3}$

$$\frac{4\pi R^3}{3} \propto A$$
$$R^3 \propto A$$

$$\boxed{R = R_0 A^{1/3}}$$

Where, R_0 is a constant and its value is $1.2 \times 10^{-15} \text{ m} = 1.2 \text{ fm}$ R -radius of a nucleus.

Nuclear Density: It is the ratio of mass and volume of nucleus.

Let A be the mass number and R be the radius of a nucleus. If m is the average mass of a nucleon then, Total Mass of nucleus = mass of each nucleon \times total number of nucleon = mA

$$\text{Volume of nucleus} = \frac{4\pi R^3}{3} = \frac{4\pi}{3} (R_0 A^{1/3})^3 = \frac{4\pi R_0^3 A}{3} \quad \{ R = R_0 A^{1/3}$$

$$\text{Nuclear density } (\rho_{\text{nu}}) = \frac{\text{mass of nucleus}}{\text{volume of nucleus}} = \frac{mA}{\frac{4\pi R_0^3 A}{3}}$$

$$\rho_{\text{nu}} = \frac{3m}{4\pi R_0^3}$$

Clearly, nuclear density is independent of Mass number A or the Size of the nucleus and therefore same for all nuclei.

$$m = 1.67 \times 10^{-27} \text{ kg} \quad R_0 = 1.2 \times 10^{-15} \text{ m, we get}$$
$$\rho_{\text{nu}} = 3 \times 1.67 \times 10^{-27} / 4 \times 3.142 \times (1.2 \times 10^{-15})^3$$
$$\rho_{\text{nu}} = 2.30 \times 10^{17} \text{ kgm}^{-3}$$

Nuclear Force: It is the strong attractive interaction acting between the nucleons that binds the proton and neutrons together inside nucleus. According to the Yukawa, the nuclear force acts between the nucleon due to continuous exchange of π -mesons between them.

Properties of nuclear force:

- 1) Short- range force act up to few fm
- 2) It shows Charge independent character.
- 3) It shows saturation effect.
- 4) It is a non-central force.
- 5) It is a non – conservative force.
- 6) It doesn't follow inverse square law.
- 7) It is the strongest interaction among basic natural forces their relative strength are, $F_g : F_e : F_n = 1:10^{36}:10^{38}$
- 8) Variations with distance - which can be understand on the basis of graph between Potential energy and separation between nucleons.

Mass defect (Δm): The difference between rest mass of a nucleus and sum of rest masses of its constituent nucleon is called mass defect.

Mass defect arises due to fact that when nucleons combine to form nucleus, some mass is converted in to binding energy in accordance with Einstein's mass-energy equivalence relation $\Delta E = \Delta mc^2$. So mass of nucleus (composite particles) is less than that of nucleons (masses of its constituents). The number of protons and neutrons are same before and after nuclear reaction only binding energy change.

Mass defect = total mass of nucleons – mass of nucleus

Mass defect = (mass of protons + mass of neutrons) – mass of nucleus

$$\Delta m = [Zm_p + (A-Z) m_n] - m_N$$

Packing fraction: Mass defect per unit mass number of an atom is known as packing fraction.

$$P.F \text{ of a nucleus} = \frac{\text{Mass defect } (\Delta m)}{\text{Mass number}(A)} \times 10^4 = \frac{(\text{Isotopic Mass} - \text{Mass Number})}{\text{Mass number}(A)} \times 10^4$$

Significance of Packing fraction:

If P.F is negative for a nucleus, the isotopic mass is less than the mass number and in that case mass gets transformed in to energy in the formation of nucleus and that element is stable for example nuclei's with Mass number 20 to 200.

If P.F is positive for a nucleus then nucleus of that element is unstable for example nuclei's with Mass number less than 20 and above 200(not necessarily true for all atoms of lower atomic mass).

Binding energy: The amount of energy required to break up a nucleus into its constituent particles, protons and neutrons and to separate far apart so that they do not interact with each other is called binding energy.

Mass defect is, $\Delta m = Zm_p + (A-Z)m_n - m_N$

From, Einstein's Mass – Energy Equivalence

$$\Delta E_b = \Delta mc^2 = [Zm_p + (A-Z)m_n - m_N] c^2 \dots\dots\dots(1)$$

The binding energy of electrons in nucleus,

$$(\Delta E_b)_e = [(m_N + Zm_e) - m(\frac{A}{Z}X)] c^2 \dots\dots\dots(2)$$

B.E. of electron is negligible in comparison to nucleon as mass of electron is very small

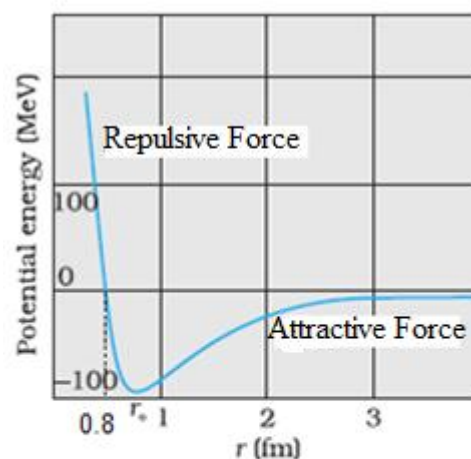


FIGURE : Potential energy of a pair of nucleons as a function of their separation. For a separation greater than r_0 , the force is attractive and for separations less than r_0 , the force is strongly repulsive.

So, $(\Delta E_b)_e = 0 \quad m\left(\frac{A}{Z}X\right) - \text{atomic mass.}$

$$m_N + Zm_e - m\left(\frac{A}{Z}X\right) = 0$$

$$m_N = m\left(\frac{A}{Z}X\right) - Zm_e$$

Putting this value equation (1),

$$\begin{aligned} \text{Thus, binding energy is given by, } \Delta E_b &= [Zm_p + (A-Z)m_n - m\left(\frac{A}{Z}X\right) + Zm_e] c^2 \\ &= [Z(m_p + m_e) + (A-Z)m_n - m\left(\frac{A}{Z}X\right)] c^2 \end{aligned}$$

$m_p + m_e = m_H$ mass of hydrogen atom.

$$\Delta E_b = [Zm_H + (A - Z)m_n - m\left(\frac{A}{Z}X\right)]c^2$$

Binding energy per Nucleon: It is the amount of energy required to extract one nucleon from the nucleus.

$$(\Delta E_b)_n = \frac{\Delta E_b}{A}$$

Binding energy curve: It is a plot of the binding energy per nucleon $(\Delta E_b)_n$ versus the mass number (A) for a large number of nuclei.

B.E. curve have following important features,

- 1) B.E curve is a smooth line for all nuclei except for He, C, O, Li, and N.
- 2) In the range mass number 2 to 20, there are well defined maxima and minima, maxima occur for ${}^2_2\text{He}^4$, ${}^6_6\text{C}^{12}$ and ${}^8_8\text{O}^{16}$ and minima for ${}^3_3\text{Li}^6$, ${}^7_7\text{N}^{14}$ and ${}^8_8\text{O}^{18}$.
- 3) The most stable nuclei is ${}^{56}_{26}\text{Fe}$ (8.8 MeV/nucleon).
- 4) From mass number 40 to 120 B.E. per nucleon increases to 8.5 MeV, further it decreases due to Coulomb's repulsion between protons.

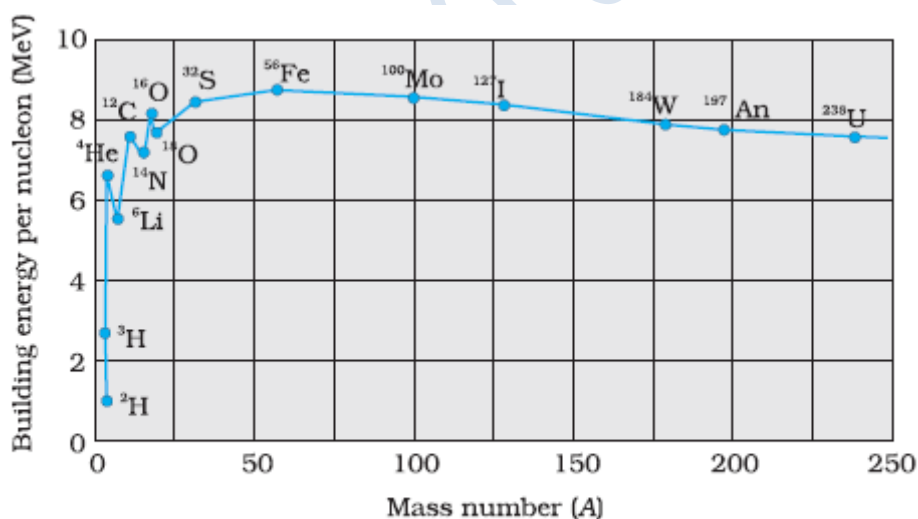


FIGURE The binding energy per nucleon as a function of mass number.

Significance of B.E curve:

Nuclear fission: B.E. per nucleon smaller than middle one for heavier nuclei ($A > 240$) so unstable, when heavy nucleus splits into the lighter nuclei energy is liberated. This is the basis of Atom Bomb.

Nuclear fusion: B.E. per nucleon is small for light nuclei ($A < 10$) so less stable. When two small nuclei combine to form heavy nuclei energy is released. This is the basis of Hydrogen Bomb.

Stability of nuclei: Criteria for stability of different nuclei are as following,

- (i) Nuclei which have high value of binding energy per nucleon are more stable.
- (ii) Neutron to proton ratio for stable nuclei should be between 1.1 to 1.6.
- (iii) Nuclei having even number of proton and neutron are more stable than nuclei having odd number except four nuclides ${}^2_1\text{H}$, ${}^6_3\text{Li}$, ${}^{10}_5\text{B}$ and ${}^{14}_7\text{N}$ which have odd number of protons and neutrons but stable called *odd-odd nuclides*.

Radioactivity: The phenomenon of spontaneous disintegration of nucleus of an atom with emission of α -particles, β -particles and γ -rays. This phenomenon was discovered by Henry Becquerel in 1896.

Radioactive decay law: The rate of disintegration (dN/dt) of a radioactive sample at any instant is directly proportional to the number of undecayed nuclei (N) present in the sample at that instant.

$$-\frac{dN}{dt} \propto N$$

$$-\frac{dN}{dt} = \lambda N$$

Where λ is Disintegration constant or decay constant

$$\frac{dN}{N} = -\lambda dt \dots\dots\dots(1)$$

Integrating above equation,

$$\int \frac{dN}{N} = -\int \lambda dt$$

$$\log_e N = -\lambda t + c \dots\dots\dots(2)$$

Finding value of integration constant, at $t = 0$, $N = N_0$ (number of undecayed nuclei at $t=0$)

Putting this value in equation in equation (2),

$$\log_e N_0 = \lambda \times 0 + c$$

So, equation (2) becomes,

$$\log_e N = -\lambda t + \log_e N_0$$

$$\log_e N - \log_e N_0 = -\lambda t$$

$$\log_e \left(\frac{N}{N_0}\right) = -\lambda t$$

$$N = N_0 e^{-\lambda t} \dots\dots\dots(3)$$

Radioactive Decay constant: It may be defined as reciprocal of time interval during which the number of active nuclei in a given radioactive sample reduces to 36.8% ($\frac{1}{e}$ times) of its initial value.

If $t = \frac{1}{\lambda}$, then

$$N = N_0 e^{-1} = \frac{N_0}{e} = \frac{N_0}{2.718} = 0.368 N_0$$

$$N = 36.8\% N_0$$

Activity or Decay rate (R): It is defined as the number of radioactive disintegration taking place per second in the sample.

$$R = -\frac{dN}{dt} = \lambda N$$

$$R = R_0 e^{-\lambda t}$$

Unit of Radioactivity:

The SI unit of Radioactivity is Becquerel (Bq).

$$1Bq = 1 \text{ decay per second} = 1s^{-1}$$

Another unit is curie (Ci) and Rutherford

$$1 \text{ curie (Ci)} = 3.7 \times 10^{10} \text{ decay per sec}$$

$$1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq}$$

$$1 \text{ rutherford} = 10^6 \text{ decay per sec}$$

Half life: The time interval in which one –half of the radioactive nuclei originally present in the radioactive sample disintegrate.

$$\text{at } t = T_{1/2}, N = N_0/2$$

Putting this value in equation (3),

$$\text{then, } N_0/2 = N_0 e^{-\lambda T_{1/2}}$$

$$1/2 = e^{-\lambda T_{1/2}}$$

$$2 = e^{\lambda T_{1/2}}$$

taking log on both side,

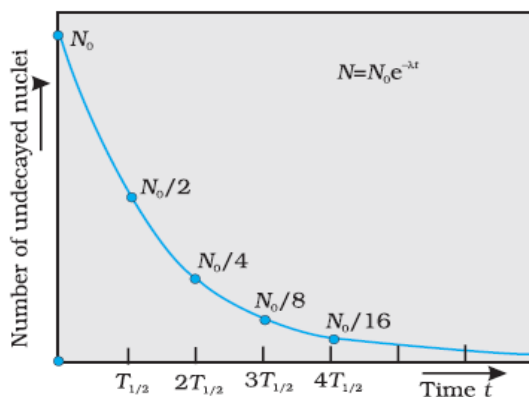


FIGURE 13.3 Exponential decay of a radioactive species. After a lapse of $T_{1/2}$, population of the given species drops by a factor of 2.

where c is integration constant

$$\lambda T_{1/2} = \log_e 2$$

$$T_{1/2} = \log_e 2 / \lambda = 2.303 \log 2 / \lambda$$

$$T_{1/2} = \frac{0.693}{\lambda}$$

Number of radioactive nuclei left undecayed after n half-life (N) = $N_0 (1/2)^n$

Total time of n half-life $t = n \times T_{1/2}$

Mean life: The average time for which the nuclei of a radioactive sample exist is called mean life or average life.

$$\text{Mean life } (\tau) = \frac{\text{sum of lives of all the nuclei}}{\text{total number of nuclei}}$$

Consider a radioactive sample contains N_0 nuclei at $t = 0$, after time t this number reduces to N , further suppose dN nuclei disintegrate in time $(t+dt) \approx t$ (as $dt \ll t$)

Total life of dN nuclei = $t dN$

Total life of all the N_0 nuclei = $\int_0^{N_0} t dN$

$$\text{Mean life } (\tau) = \frac{\text{sum of lives of all the nuclei}}{\text{total number of nuclei}} = \frac{\int_0^{N_0} t dN}{N_0}$$

From equation (3)

$$N = N_0 e^{-\lambda t}$$

Differentiating above equation,

$$dN = -\lambda N_0 e^{-\lambda t} dt$$

Changing limit, when $N = N_0$ then $t = 0$ and $N = 0$ then $t = \infty$

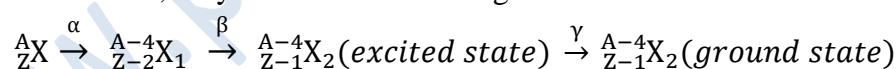
$$\tau = \frac{\int_0^{\infty} t(-\lambda N_0 e^{-\lambda t}) dt}{N_0}$$

$$\tau = \lambda \int_0^{\infty} t e^{-\lambda t} dt$$

$$\tau = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693} = 1.44 T_{1/2}$$

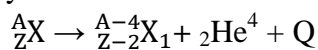
Fajan's and Soddy law/Displacement laws for radioactive transformation:

- 1) When a radioactive nucleus emit an α -particle, its atomic number decrease by 2 and mass number decrease by 4.
- 2) When a radioactive nucleus emits a β -particle, its atomic number increase by 1 but mass number remains the same.
- 3) The emission of a γ -particle does not change the mass number or the atomic number of the radioactive nucleus, only nuclei converted to ground state from excited state.



α -Decay: It is the phenomenon of emission of an α -particle (${}_2\text{He}^4$) from a radioactive nucleus.

In α -decay, the mass number of the product nucleus is four less than that of decaying nucleus, while the atomic number decreases by two.



Generally α -particle emission occur from nuclei which have mass number more than 210, in such nuclei long range electrostatic force is dominates over short range nuclear forces. An α -particle have large amount of binding energy (28MeV), So after emission of an α -particle binding energy per nucleon increases and nuclei becomes more stable.

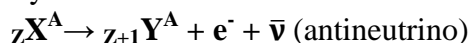
K.E. of α -particle emitted in α -decay,

$$K_\alpha = \frac{A-4}{A} \cdot Q$$

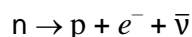
Q – Amount of energy released in α -decay process

β -Decay: It is the phenomenon of emission of an electron (e^-) or a positron (e^+) from a radioactive nucleus. In β -decay, the mass number of the product nucleus is same but atomic number increases or decreases by one.

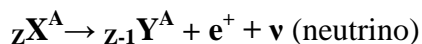
✚ The β^- decay an electron alongwith a new particle antineutrino ($\bar{\nu}$) are emitted and atomic number increases by one.



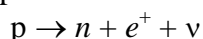
The basic nuclear process underlying β^- decay is the conversion of neutron to proton



✚ While in β^+ decay a positron (e^+ -anti-particle of electron) alongwith a neutrino (ν) are emitted and atomic number decreases by one.



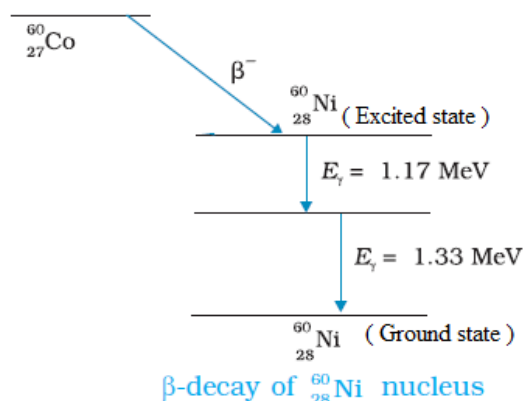
While for β^+ decay, it is the conversion of proton into neutron



Neutrinos and *antineutrinos* are neutral/chargeless particles with very small (possibly, even zero) mass compared to electrons. They have only weak interaction with other particles. They are, therefore, very difficult to detect, since they can penetrate large quantity of matter (even earth) without any interaction.

In both β^- and β^+ decay, neutron converted in to proton and vice-versa so, mass number A of nuclei remains unchanged as mass of proton is nearly equal to mass of neutron.

γ -Decay: A γ -ray is emitted when an α or β decay results in a daughter nucleus in an excited state. This then returns to the ground state by a single photon transition or successive transitions involving more than one photon.



PROPERTY	α -Rays	β -Rays	γ -Rays
Identity	It is a beam of helium nucleus (${}^4_2\text{He}$)	It is a beam of fast moving electrons (${}_{-1}e^0$)	It is an electromagnetic radiation in which high energy photons are Emitted.
Charge	+2e	-e	No charge
Rest mass	4amu	Equal to mass of electron	Zero
Speed	approximately $0.1C$ (10^7ms^{-1})	$0.33C$ to $0.99C$ (depending on radioactive nuclide)	C ($3 \times 10^8\text{ms}^{-1}$)
Deflection by E.Field & M.field	Deflected by electric and magnetic fields	Deflected by electric and magnetic fields	No Deflection by electric and magnetic fields.
Penetration range	Stopped by a sheet of paper A few cm in air (because of large mass)	Several millimeter in plastic Several metres in air	Very large can penetrate through several cm lead
Fluorescence	Show fluorescence (on ZnS, barium plantinocyanide)	Show fluorescence(on willmite, barium platinocyanide)	Show fluorescence (on willmite)
Effect on photographic plates	Affects photographic (maximum)	Affects photographic (moderate)	Affects photographic (minimum)
Ionising power	Highly ionizing (because of large mass)	Less than α -rays $\left\{ \frac{1}{100} \text{th of } \alpha\text{-rays} \right\}$	Small ionizing power $\left\{ \frac{1}{10,000} \text{th of } \alpha\text{-rays} \right\}$

Question: Why α - ray have more ionising power than β -Ray?

Answer: The speed of α -particles is less than that of β -particles. When α -particles are allowed to pass through gaseous medium, then due to their less speed they remain in contact with atoms of the gas for

more times as compared to the β -particles. So α -particles are able to transfer sufficient energy for ionisation of gaseous atoms whereas the β -particles being faster cannot do so. Thus, ionisation power of α -particles is more than that of β -particles.

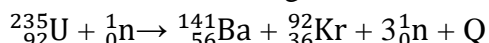
Question: Why β – ray have more penetration power than α ?

Answer: While penetrating a substance, α - particles dissipate more energy in lesser distance due to their small velocities. Thus, their energy lasts for small distance. On the other hand β -particles spend less energy in the same distance and their energy lasts for greater distance.

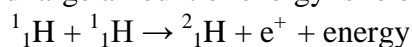
Nuclear energy: The energy which is obtained from nuclear reactions is called nuclear energy.

Nuclear Reaction:

- 1) Nuclear fission: The phenomenon in which a heavy nucleus ($A > 230$) splits into two smaller nuclei and large amount of energy is released, is called nuclear fission.



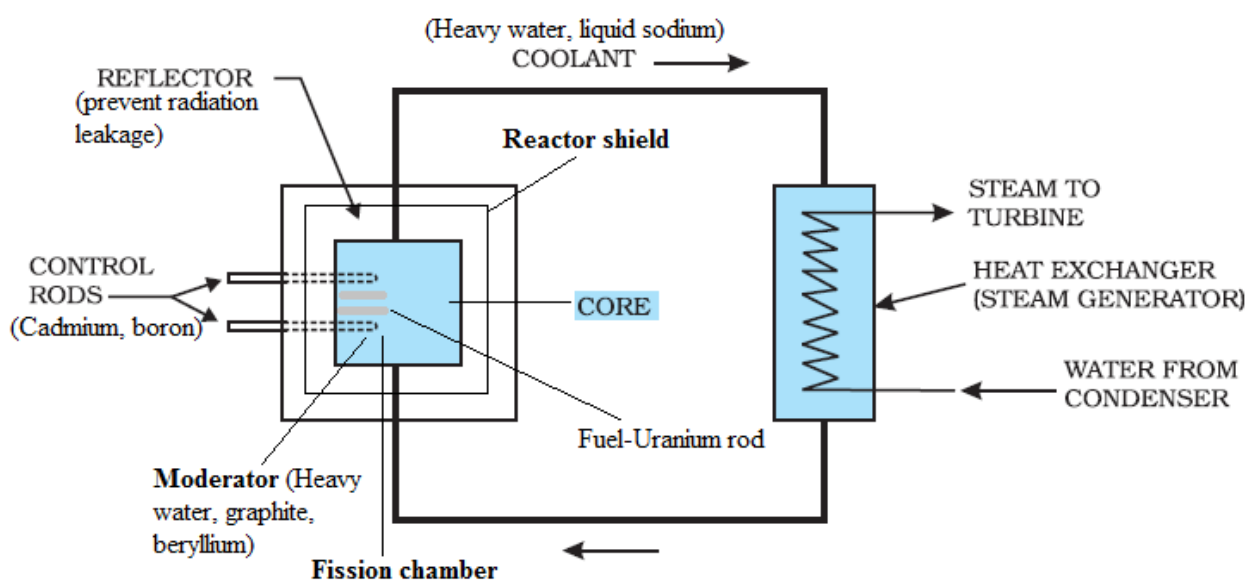
- 2) Nuclear fusion: The phenomenon in which two light nuclei combine to form a single heavy nucleus and large amount of energy is released, is called nuclear fusion.



In any nuclear reaction total momentum, number of nucleons, total charge and total mass-energy is conserved.

Nuclear chain reaction: The nuclear reaction in which number of fissions taking place at each successive stage goes on increasing at a rapid rate (in a geometric progression) is called nuclear chain reaction.

Nuclear reactor: It is a device in which nuclear chain reaction initiated, maintained and controlled and large amount of energy is produced.



Schematic diagram of a nuclear reactor based on thermal neutron fission.

Nuclear fuel: Mostly ${}_{92}^{235}\text{U}$ and ${}_{94}^{239}\text{Pu}$ are used as nuclear fuel.

Moderator: It slow down speed of neutrons (such neutrons are called *thermal neutrons*). *Heavy water is ideal for moderator* because when a fast moving neutron collides with heavy-hydrogen nuclei (deuterium) it gets slowdown as collision between both is elastic and their masses are nearly equal.

Mass of light-hydrogen nuclei (protium) is also comparable to neutron but these nuclei have greater probability of capturing neutrons rather than slowdown them and while heavy water have negligible cross-section for neutron capture so it slowdown neutron.

Control rods: It absorbs excess neutrons and control nuclear chain reaction at constant rate.

Coolant: It absorbs heat energy released by nuclear reaction and transfers heat to heat exchanger.

Liquid sodium and heavy water are commonly used as coolant.

Difficulties in sustaining a Nuclear Chain Reaction: Natural occurring uranium consists of three isotopes ${}^{234}_{92}\text{U}$ (0.006%), ${}^{235}_{92}\text{U}$ (0.715%) and ${}^{238}_{92}\text{U}$ (99.28%), out of which ${}^{235}_{92}\text{U}$ is required for sustaining nuclear chain reaction. So concentration of ${}^{235}_{92}\text{U}$ is increased by a special technique called **enrichment**, enriched uranium consists of about 3% of ${}^{235}_{92}\text{U}$. Some other difficulties are neutron leakage (overcome by reducing surface to volume ratio), high neutron energy (slowing down them by using moderators).

Multiplication factor (k): It is the ratio of number of neutrons present at the beginning of a particular generation to the number of neutrons present at the beginning of the previous generation.

$$k = \frac{\text{number of neutrons present at the beginning of one generation}}{\text{number of neutrons present at the beginning of previous generation}}$$

Significance of Multiplication factor,

If $k > 1$ - Nuclear chain reaction grows

$k = 1$ - Nuclear chain reaction remains steady

$k < 1$ - Nuclear chain reaction gradually dies out.

Critical Size and Critical Mass: For a nuclear chain reaction to sustain the size or the mass of fissionable material should be larger than its critical value, If not than nuclear chain reaction can't be sustain that critical value of size of fissionable material is known as critical size and mass is known as critical mass. This fact is used by Nuclear Energy Institutions to store nuclear fuel. For critical size and critical mass multiplication factor is 1.

Questions for practice:

- Name the absorbing material used to control the reaction rate of neutrons in a nuclear reactor. (01)
- The mass of H-atom is less than the sum of the masses of a proton and electron. Why is this? (01)
- Draw a graph showing the variation of decay rate with number of active nuclei? (01)
- He^3_2 and He^3_1 nuclei have the same mass number. Do they have the same binding energy? (01)
- The three stable isotopes of neon ${}^{20}_{10}\text{Ne}$, ${}^{21}_{10}\text{Ne}$ and ${}^{22}_{10}\text{Ne}$ have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of three isotopes are 19.99u, 20.99u and 21.99u respectively. Obtain the average atomic mass of neon. (01)
- What is the shortest frequency present in the Paschen series of spectral lines? (01)
- Draw a plot of potential energy of a pair of nucleons as a function of their separation. Write two important conclusions which you can draw regarding the nature of nuclear forces on the basis of it. (02)
- Using postulates of Bohr's theory of hydrogen atom show that the radius of orbits increases as n^2 , where n is the principal quantum number of the atom. (02)
- The energy of the electron in the ground state of hydrogen atom is -13.6eV .
 - What does the negative sign signify?
 - How much energy is required to take an electron in this atom from the ground state to the first excited state? (02)
- How the size of a nucleus is experimentally determined? Show that density of nucleus is independent of its mass number. (02)
- Derive the law of radioactive decay, viz. $N=N_0e^{-\lambda t}$. (03)
- (a) What is meant by Decay constant?

- (b) The half life of a radioactive substance is 30s. Calculate Time taken for the sample to decay by $\frac{3}{4}$ of the initial value. **(03)**
13. Calculate binding energy/nucleon of $^{40}_{20}\text{Ca}$ nucleus. $^{40}_{20}\text{Ca}$ mass = 39.962589 amu, $m_p = 1.007825$ amu, $m_n = 1.008665$ amu. **(02)**
14. Draw the general shape of the plot of the binding energy per nucleon versus the mass number for different nuclei. Hence explain why we must expect a release of nuclear energy during (i) nuclear fission, (ii) nuclear fusion. **(03)**
15. Find the Q-value and the kinetic energy of the emitted α -particle in the α -decay of $^{226}_{88}\text{Ra}$. $(^{226}_{88}\text{Ra})m = 226.02540$ amu, $(^{222}_{86}\text{Rn})m = 222.01750$ amu, $(^{220}_{86}\text{Rn})m = 220.01137$ amu **(02)**
16. The half life of $^{238}_{92}\text{U}$ against α -decay is 4.5×10^9 years. How many disintegration per second occurs in 1g of $^{238}_{92}\text{U}$? **(02)**
17. Show that the speed of an electron in the innermost orbit of H-atom is $1/137$ times the speed of light in vacuum. **(02)**
18. What is the energy level diagram for an atom? Calculate the energies of the various energy levels of a hydrogen atom and draw an energy level diagram for it. **(02)**
19. The half life of $^{238}_{92}\text{U}$ against α -decay is 4.5×10^9 years. How many disintegrations per second occur in 1g of $^{238}_{92}\text{U}$? **(02)**
20. State Bohr's postulates for explaining the spectrum of hydrogen atoms. **(03)**
21. Define the terms excitation and ionization energies; and excitation and ionization potentials. **(02)**
22. Using postulates of Bohr's theory of hydrogen atom, show that (a) The total energy of the electron increases as $1/n^2$, where n is the principal quantum number of the atom. (b) Calculate the wavelength of H_α line in Balmer series of hydrogen atom, given Rydberg constant $R = 1.0947 \times 10^7 \text{ m}^{-1}$. **(03)**
-