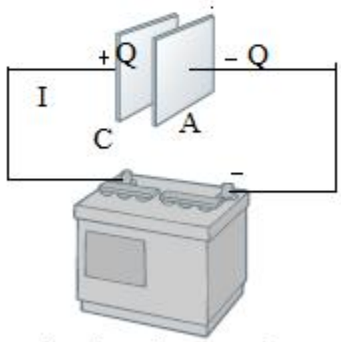
Unit – 05 Electromagnetic waves

Conduction current: The current carried by conductors due to flow of electrons.

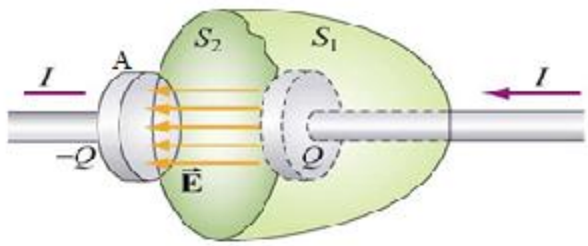
Displacement current (I_d): *The current which arises due to **changing electric field** between plates of capacitor is called displacement current*.

Consider a capacitor C is being charged by a battery amount of current flowing through capacitor is *I*. Draw closed loop S_1 and S_2 close to capacitor plate. Using Ampere's circuital law magnetic field at S_1 is given by

$$\oint \vec{B} \cdot \vec{dl} = \mu_o I_{enclosed} \qquad \ldots\ldots\ldots\ldots\ldots(1)$$



Charging of a capacitor



Changing electric field between plates of capacitor

Now apply Ampere's circuital law for second face S_2 of loop, as no conduction current flow between the plates of capacitor so magnetic field at S_2 should be equal to zero.

$$\oint \vec{B} \cdot \vec{dl} = 0 \qquad \ldots\ldots\ldots\ldots\ldots(2)$$

Since both loop S_1 and S_2 are very close to each other so equation 1 should be equal to equation 2, but both are not equal so there is a contradiction arises. This contradiction arises by using Ampere's circuital law, so Ampere's circuital seems to be inconsistent in this case i.e. this law must be missing something.

Maxwell's find this missing term, that there is variable electric field passes through the space between plates of capacitor due to which a current is produced called *displacement current*.

Time varying electric field between plates of capacitor is given by,

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{\sigma}{\epsilon_\circ}\right) = \frac{d}{dt}\left(\frac{Q}{A\epsilon_\circ}\right) = \frac{1}{A\epsilon_\circ}\frac{dQ}{dt}$$

$$\frac{dQ}{dt} = A\epsilon_\circ \frac{dE}{dt} = \epsilon_\circ \frac{d(EA)}{dt} = \epsilon_\circ \frac{d(\phi_E)}{dt}$$

$$I_d = \frac{dQ}{dt} = \epsilon_\circ \frac{d(\phi_E)}{dt}$$

Maxwell modified Ampere's circuital law as

$$\oint \vec{B} \cdot \vec{dl} = \mu_o \epsilon_o \frac{d\phi_E}{dt} + \mu_o I_{conduction}$$ ***………Modified form of Ampere's circuital law***

Properties of Displacement Current:

(i) Displacement current exists when there is a change of electric flux. Unlike conduction current, it does not exist under steady conditions.

(ii) It is not a current. It only adds to current density in Ampere's Circuital law. As it produces magnetic field, so called current.

Question: Show that displacement current is equal to conduction current.

Answer: As displacement current is given by,

$$\mathbf{I_d} = \epsilon_\circ \frac{d(\phi_E)}{dt}$$

$$\phi_E = \mathbf{EA} = \left(\frac{Q}{A\epsilon_\circ}\right)\mathbf{A} = \frac{Q}{\epsilon_\circ}$$

$$\mathbf{I_d} = \epsilon_\circ \frac{d(\phi_E)}{dt} = \epsilon_\circ \frac{d}{dt}\left(\frac{Q}{\epsilon_\circ}\right) = \frac{dQ}{dt} = \mathbf{I_C}$$

Maxwell's Equations: Maxwell's found that all the basic principles of electromagnetism can be formulated in terms of four fundamental equations called Maxwell's equation. Maxwell's equation showed relationship between electric and magnetic field and the relation of electric and magnetic field to the accelerating charge and current. *Maxwell's theory showed that **accelerating charge** produces electromagnetic wave and the frequency of wave is equal to the frequency of oscillating charge.*

(1) $\oint \vec{E} \cdot \vec{dA} = \frac{q_{enc}}{dt}$ …….**Gauss law of electrostatics**

(2) $\oint \vec{B} \cdot \vec{dA} = 0$ …….**Gauss law of magnetism**

(3) $\oint \vec{E} \cdot \vec{dl} = -\frac{d\phi_B}{dt} = \varepsilon$ …….**Faraday's law of electromagnetic induction**

(4) $\oint \vec{B} \cdot \vec{dl} = \mu_o \epsilon_o \frac{d\phi_E}{dt} + \mu_o I_{conduction}$ ……. **Modified form of Ampere's circuital law**

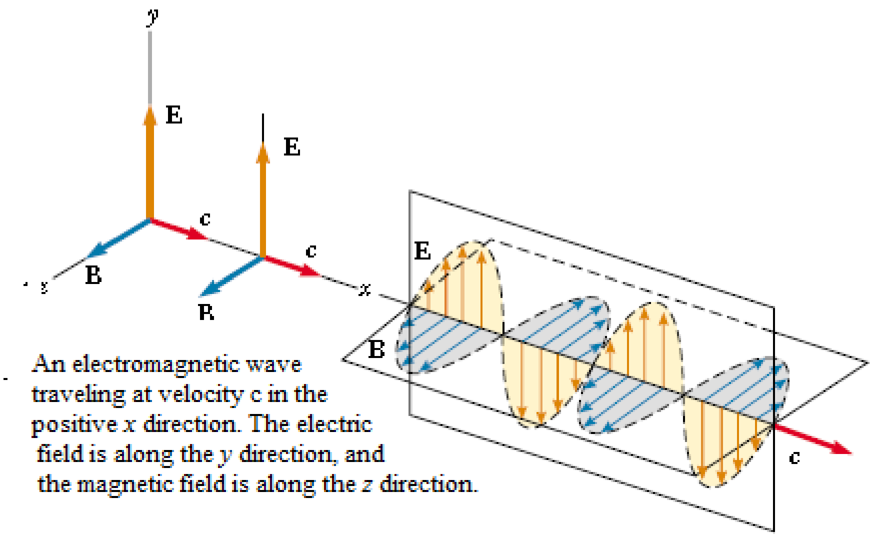
(1) Gauss's law: Gauss's law states that, the total electric flux passing through any closed surface is equal to $1/\epsilon_o$ times charge enclosed by the surface. It *relates the electric flux through an area with electric charges*. Coulomb's law can be derived from Gauss's law, hence it can be considered to *fundamental equation of electrostatics.*

(2) Gauss's law in magnetism: Gauss's law in magnetism states that the total magnetic flux, through any closed surface is always equals to zero. *It tells that magnetic field lines always forms closed loop and magnetic monopoles does not exist.*

(3) Faraday's law of Electromagnetic Induction: Faraday's law states that, the magnitude of induced electromotive force in any closed circuit is equal to rate of change of the magnetic flux through the circuit. *It tells that changing magnetic flux induces an electric field*

(4) Modified Ampère's Circuital law: The line integral of the magnetic field around any closed circuit is equal to μ_o times the total current (the sum of conduction and displacement currents) flowing the close circuit.

Electromagnetic Waves: A wave which is produced by accelerated charge in which electric field and magnetic field oscillating perpendicular to each other and direction of propagation of wave.



An electromagnetic wave traveling at velocity c in the positive x direction. The electric field is along the y direction, and the magnetic field is along the z direction.

Properties of Electromagnetic Waves:

1) At any instant ratio of electric field and magnetic field may be related as $\mathbf{c} = \frac{E_0}{B_0}$

2) Speed of an electromagnetic wave depends on permeability and permittivity of the medium $\mathbf{c} = \frac{1}{\sqrt{(\mu_0 \epsilon_0)}}$.

3) Energy density of an electromagnetic wave is equal to sum of magnetic energy and electric energy ($U = U_E + U_B$) and magnetic energy is equal to electric energy.

4) The e.m. wave is transverse in nature i.e. both electric and magnetic fields are oscillating perpendicular to each other and perpendicular to the direction of wave propagation.

5) Electromagnetic waves do not require any medium for their propagation.

6) Electromagnetic waves are uncharged hence cannot be deflected by magnetic and electric field.

7) Electromagnetic waves do not require any medium for their propagation and their speed in vacuum is 3×10^8 m/s.

8) Momentum of an electromagnetic wave P = U/c

9) Electromagnetic waves show reflection, Refraction, interference, diffraction and polarization.

10) Electromagnetic waves carry energy and rate of flow of energy crossing a unit area is described by the Poynting vector S, where

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

11) In electromagnetic wave field magnitudes of E and B vary with x and t according to the expressions

$E = E_{max} \cos(kx - \omega t)$

$B = B_{max} \cos(kx - \omega t)$

Q. Show that average energy density of Electric field is equal to average energy density of Magnetic field in e.m. waves.

Answer: In electromagnetic waves,

$$\text{Energy density of electric field } u_E = \frac{1}{2} \epsilon_o E^2$$

$$\text{Energy density of magnetic field } u_B = \frac{1}{2\mu_o} B^2$$

Now, $u_E = \frac{1}{2}\epsilon_o E^2 = \frac{1}{2}\epsilon_o (c\,B)^2$ $\{\ E = c\,B\ \}$

$u_E = \frac{1}{2}\epsilon_o E^2 = \frac{1}{2}\epsilon_o\ c^2B^2 = \frac{1}{2}\epsilon_o\ (\frac{1}{\sqrt{(\mu_o\,\epsilon_o)}})^2 B^2$ $\{\ c = \frac{1}{\sqrt{(\mu_o\,\epsilon_o)}}\ \}$

$$u_E = \frac{1}{2}\epsilon_o \frac{1}{\mu_o\,\epsilon_o} B^2 = \frac{1}{2\mu_o}\ B^2 = u_B$$

$$\boldsymbol{u_E = u_B}$$

Hertz's experiment: Hertz produces electromagnetic waves experimentally in 1887, using LC circuit. *Transmitter* consists of capacitor made up of two large square metal plates with sides 60cm x 40cm connected to two highly polished brass sphere (S_1, S_2) and secondary terminal of induction coil by thick copper wire. A source of high voltage (induction coil) is connected with copper wires, due to it a high potential difference is setup across S_1 and S_2.this high potential ionizes air in the gap the electrons/ions so produced oscillates back and forth and electromagnetic wave is produced in air gap between two spheres.

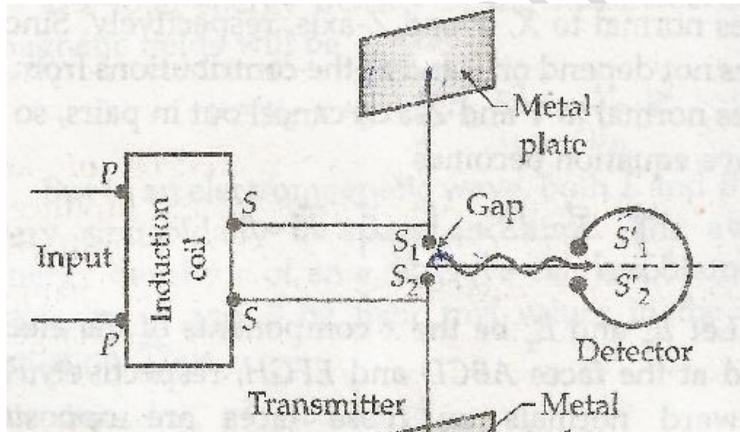


fig.Hertz's experimental setup to produces electromagnetic waves

The frequency wave produced here is given by

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Electromagnetic waves produced here can be detected by *Detector.* It is a circular wire which both ends have two small polished brass sphere, e.m. waves striking on sphere produces a spark across the air gap.

Hertz demonstrated various properties of electromagnetic wave

(i) Electromagnetic waves are transverse in nature.

(ii) Electromagnetic radiation has a wave nature.

(iii) Electromagnetic waves travels with speed of light.

(iv) Frequency of electromagnetic wave is given by $f = \frac{1}{2\pi\sqrt{LC}}$

Questions for practice:

1. Answer the following questions:

a) Long distance radio broadcasts use short-wave bands. Why?

Ans: Radio broadcast use the reflection of transmitted waves through ionosphere (a atmosphere layer up to a height of 65-400 km) this layer reflect short wave bands.

b) It is necessary to use satellites for long distance TV transmission. Why?

Ans: TV signals are not properly reflected by ionosphere. Therefore, signals are made to be reflected to earth using satellites.

c) Optical and radio telescopes are built on the ground but X-ray astronomy is possible only from satellites orbiting the earth. Why?

Ans: The visible radiations and radiowaves can penetrate the earth's atmosphere but X-ray is absorbed by atmosphere.

d) If the earth did not have an atmosphere, would its average surface temperature be higher or lower than what, it is now?

e) The small ozone layer on top of the stratosphere is crucial for human survival. Why?

f) Some scientists have predicted that a global nuclear war on the earth would be followed by a severe 'nuclear winter' with a devastating effect on life on earth. What might be the basis of this prediction?

2. The oscillating electric field of an electromagnetic wave is given by: $E_y = 30 \sin [2 \times 10^{11} t + 300\pi x]$ Vm^{-1}. (i) Obtain the value of the wavelength of the electromagnetic wave. (ii)Write down the expression for the oscillating magnetic field.

Ans: λ= 2/3 cm ,$B_Z = 10^{-7}\sin (2 \times 10^{11} t\ 300\ \pi x)$ T

3. The oscillating magnetic field in a plane electromagnetic wave given by $B_Y = 8X10^{-4} (2 x10^{11} t + 300\pi x)$ tesla.(i) Calculate the wavelength of electromagnetic wave. (ii)Write down the expression for oscillating electric field. **Ans**: λ= 6.67 x 10^{-3}m, E_Z= 2.4 x $10^3 \sin(2x10^{11}t +300\ \pi x)Vm^{-1}$

4. Why does microwave oven heats up a food item containing water molecules most efficiently?
Answer: In a microwave ovens, the frequency of the microwaves must match the resonant frequency of the water molecules so that energy from the waves is transferred efficiently the kinetic energy of the molecules. This increases the temperature of the food item sufficiently.

5. Why is the orientation of the portable radio with respect to broadcasting station important?
Answer: Signals transmitted from broadcasting station is in the form of plane polarized electromagnetic waves. These waves are received by radio if the receiving antenna is parallel to either electric field or magnetic field of the wave.

6. A variable frequency a.c source is connected to a capacitor. How will the displacement current change with decrease in frequency?

7. How do you show that electromagnetic waves carry energy and momentum?
Answer: When a charge oscillates with some frequency it produces an oscillating electric and magnetic field in space and hence an electromagnetic wave is produced. The frequency of em wave is equal to the frequency of oscillation of the charge as energy associated with em wave comes at the expense of the source.
If em wave of energy U strikes on a surface and gets completely absorbed, total momentum delivery to the surface is p = U/E . Hence em wave carry momentum also.

8. You are given a 2μF parallel plate capacitor. How would you establish an instantaneous displacement current of 1mA in the space between its plates?

9. Light with an energy flux of 18 watts / cm^2 falls on a non – reflecting surface at normal incidence. If the surface has an area of $20cm^2$, find the average force exerted on the surface during a 30 minute time span.
Ans: 1.2 x 10^{-6} N

10. Shows a capacitor made of two circular plates each of radius 12 cm and separated by 5.0 mm. the capacitor is being charged by an external source. The charging current is constant and equal to 0.15 A. (i) Calculate the capacitance and the rate of change of potential difference between the plates. (ii) Obtain the displacement current across the plates. (iii) Is Kirchhoff's first law (junction rule) valid at each plate of the capacitor?
Ans: i) C = 80.1pF, 1.875 x $10^9 Vs^{-1}$ ii) I = 0.15A iii) yes

11. A parallel plate capacitor made of circular plates each of radius R = 60cm has a capacitance C = 100pF. The capacitor is connected to a 230V a.c. supply with a (angular) frequency of 300 rad s^{-1}.

a) What is the rms value of the conduction current?
b) Is the conduction current equal to the displacement current?
c) Determine the amplitude of Bat a point 3.0 cm from the axis between the plates.

Ans: a) i = 6.9μA b) yes, the conduction current is equal to the displacement current even if I is oscillating in time. c) 1.63 x 10^{-11}T.

12. Calculate the electric and magnetic fields produced by the radiation coming from a 100 watt bulb at a distance of 3 m. Assume that the efficiency of the bulb is 2.5% and it is a point source.

Ans: E_o= 4.1 Vm^{-1}, B_o= 1.4 x 10^{-8} T

13. How does a charge q oscillating at certain frequency produce electromagnetic waves? How is the frequency of the electromagnetic waves produced related to that of the oscillating charge?

Answer: An oscillating charge produces an oscillating Electric field. Oscillating electric field produces an oscillating magnetic field. E and B varying with time regenerates each other as a result e.m. waves is produced.

Frequency of electromagnetic wave is equal to the frequency of oscillating charge.

14. When an ideal capacitor is charged by a DC battery, no current flows. However, when an AC source is used, the current flows continuously. How does one explain this, based on the concept of displacement current?

Answer: Displacement current establishes continuity of current between the capacitor plates. Displacement current is caused by changing electric flux between the plates of capacitor.

$$\mathbf{I_d = \epsilon_\circ \frac{d(\phi_E)}{dt}}$$

In case of direct current there is no change in electric flux, i.e. there is no displacement current. As a result there being no continuity, capacitor does not conduct.

For alternating current source voltage between the capacitor plates keeps on changing.

$$\mathbf{V = Ed}$$

So, electric field keeps on changing accordingly. Changing electric field makes the flux change. Hence, displacement current is produced and capacitor conducts.

15. Electromagnetic waves with wavelength, (i) λ_1 is used in satellite communication (ii) λ_2 is used to kill germs in water purifies (iii) λ_3 is used to detect leakage of oil in underground pipelines (iv) λ_4 is used to improve visibility in runways during fog and mist conditions.

(a) Identify and name the part of electromagnetic spectrum to which these radiations belong.
(b) Arrange these wavelengths in ascending order of their magnitude.
(c) Write one more application of each.